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Massive Quantum Vortex Excitations in a Pure Gauge Abelian Theory in 2+1D

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Abstract

We introduce and study a pure gauge abelian theory in 2+1D in which massive quantum vortex states do exist in the spectrum of excitations. This theory can be mapped in a three dimensional gas of point particles with a logarithmic interaction, in the grand-canonical ensemble. We claim that this theory is the 2+1D analog of the Sine-Gordon, the massive vortices being the counterparts of Sine-Gordon solitons. We show that a symmetry breaking, order parameter, similar to the vacuum expectation value of a Higgs field does exist.

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1) Introduction

Massive vortex excitations usually occur in gauge theories coupled to a symmetry breaking Higgs field in the broken phase [1]. In this work, we introduce a three dimensional pure gauge abelian theory which presents a mechanism for the generation of mass for quantum vortex excitations that does not require the presence of a Higgs field. This is completely analogous to the one through which the Sine-Gordon solitons acquire a mass in two space-time dimensions [2]: if we turn off the cossine interaction in the Sine-Gordon theory, we are left with a free massless field, in which the soliton operator creates massless excitations [2]. The quantum "soliton" system can then be associated to an electrostatic system of point charges such that the "soliton" correlation functions are exponentials of the electrostatic energy of these charges [3]. When we turn on the cossine interaction, these external charges associated with the solitons are imbibed in a gas of point (dual) charges in the grand-canonical ensemble [4] and the effective interaction of the former charges is thereby modified, leading to a behavior of the correlation functions which indicates the solitons become massive.

We start by considering the vortex operator [5, 6, 7] in a modified Maxwell theory in 2+1D (described by (2.6)) which is the three dimensional analog of the free massless scalar field, in the sense that its quantum vortex states are massless. It is rather suggestive that this theory has appeared for the first time in the bosonization of the free massless Dirac fermion field in 2+1D [8] in analogy to its two dimensional counterpart [9]. In this framework, the vortex/soliton states are related to the associated free fermions. This modified Maxwell theory also appears as the effective three dimensional theory describing the real electromagnetic interaction experienced by four dimensional charged particles constrained to move on an infinite plane [10].

In order to obtain massive vortex states, we consider an operator which is dual to the vortex operator [11] and construct a gas containing the point charges created by it in the grand-canonical ensemble. A new theory is thereby generated, whose vacuum functional is the grand-partition function of the gas. In this theory, which we call "Sine-Maxwell", in analogy to the Sine-Gordon, the charge bearing dual operator has

a nonzero vacuum expectation value, resembling what happens when the gauge theory is coupled to a symmetry breaking Higgs field. Of course, one immediately would ask whether a mass would be dynamically generated to the gauge field in this theory. In Section 4 we explain that further investigation is required in order to answer this question.

We evaluate the vortex two-point function in the above described theory using a fugacity expansion for the gas. Comparing the resulting series with the corresponding one for the Sine-Gordon solitons we can immediately infer the massiveness of the quantum vortices.

2) Vortex Operators

Quantum vortex creation operators in 2+1D were introduced years ago in theories containing an abelian vector gauge field [5, 6, 7]. In a pure Maxwell theory, namely,

$$\mathcal{L}_M = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{2.1}$$

it is given by [7, 12]

$$\mu(x)\exp\left\{-ib\int_{T_{L(x)}} d^2\xi\partial_i arg(\xi - x)F^{i0}(\xi, x^0)\right\} \tag{2.2}$$

or

$$\mu(x;T_{L(x)}) = \exp\left\{-\frac{i}{2}\int d^3zA^{\mu\nu}F_{\mu\nu}\right\} \tag{2.3}$$

where the external field $A^{\mu\nu}$ is given by

$$A_{\mu\nu}(z;x) = b\int_{T_{L(x)}} d^2\xi_{[\mu}\partial_{\nu]}\arg(\vec{\xi} - \vec{x})\delta^3(z - \xi) \tag{2.4}$$

Where $T_{L(x)}$ is the surface represented in Fig.1. Even though μ depends explicitly on the surface $T_{L(x)}$, it can be shown [6, 7] that its renormalized correlation functions are local. The vortex operator μ creates quantum states carrying $2\pi b$ units of flux. In this trivial case the renormalized quantum vortex correlation functions are [7, 12]

$$< \mu\mu^\dagger >_R = \exp\left\{\frac{\pi b^2}{|x - y|}\right\} \tag{2.5}$$

The large distance behavior indicates that $<\mu>_R\neq 0$ and therefore the μ-operator does not create genuine vortex excitations in pure Maxwell theory as one would expect.

A nonlocal generalization of Maxwell theory given by

$$\mathcal{L}_{MNL} = -\frac{1}{4}F^{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu} = -\frac{1}{4}\int d^3y F^{\mu\nu}(x)K(x-y)F_{\mu\nu}(y) \tag{2.6}$$

has been studied recently in connection with the bosonization of a free massless Dirac fermion field in 2+1D [8]. This theory was also shown to describe the actual electrodynamic interaction of charged particles constrained to move on an infinite plane [10]. In (2.6)

$$K(x-y) = \left[\frac{1}{(-\Box)^{1/2}}\right] = \int \frac{d^3k}{(2\pi)^3}\frac{e^{ik\cdot(x-y)}}{(k^2+i\epsilon)^{1/2}} = \frac{i}{2\pi\left[(x-y)^2+i\epsilon\right]} \tag{2.7}$$

The quantization of theories of the type given by (2.6) has been studied carefully in [13]. In spite of the nonlocality, they are shown to respect causality and to be perfectly well defined. In the specific case of (2.6), the Green functions are shown to have support on the light-cone surface [13], a fact which makes the theory to obey the Huygens principle in analogy to the four dimensional Maxwell theory, a property which is not shared by its three dimensional counterpart.

In this case, the vortex operator μ is given by [8, 11]

$$\mu(x;T_{L(x)}) = \exp\left\{-\frac{i}{2}\int d^3z A^{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}\right\} \tag{2.8}$$

where $A^{\mu\nu}$ is still given by (2.4). The renormalized correlation function of μ in the theory described by (2.6) has been evaluated in [11]. The result is

$$<\mu(x)\mu^\dagger(y)>_R = \frac{1}{|x-y|^{b^2}} \tag{2.9}$$

The large distance behavior of (2.9) now indicates that $<\mu>=0$ and therefore there are true quantum vortex excitations in this theory. The power-law decay, on the other hand, is an evidence that these states are massless. These massless vortex states are the ones that are associated to the massless fermion field in the process of bosonization introduced in [8]. The situation is analogous to the one occuring in 1+1D when the

soliton operator introduced in the theory of a massless scalar field creates massless states which correspond to the massless fermion field [2]. The theory described by (2.6) is the three dimensional analog of the 1+1D massless scalar field. In order to obtain a theory with nontrivial massive vortex excitations what is done in the two dimensional case is that an order operator dual to μ is introduced [3] and a theory is constructed whose vacuum functional is the grand partition function of a gas of the excitations created by this order parameter [4]. The resulting theory is the familiar Sine-Gordon [2]. In the following Section, we are going to perform the analogous steps in 2+1D. We will start from (2.6) and consider the order operators dual to μ which were introduced in [8, 11]. Then, we will construct a gas with the excitations created by these operators and eventually arrive at a partition function that is associated to a theory which is the three dimensional counterpart of the Sine-Gordon.

3) The "Sine-Maxwell" Theory

3.1) The σ Operator and Its Correlation Functions

The σ operator which is dual to μ in the nonlocal Maxwell theory described by the lagrangian (2.6) was introduced and studied in [8, 11]. Similarly to μ, we can also write it in terms of an external field, which in this case is given by

$$\tilde{C}_{\mu\nu}(z;x) = a \int_{x,L}^{\infty} d\xi_\mu \partial_\nu \left[\frac{1}{(-\Box)^{1/2}} \right] (z-\xi) \tag{3.1}$$

We can express σ as a functional of $\tilde{C}_{\mu\nu}$ as [11]

$$\sigma(x) = \exp \left\{ -i \int d^3 z \tilde{C}_{\mu\nu} \left[\frac{1}{(-\Box)^{1/2}} \right] F^{\mu\nu} \right\}$$

or

$$\sigma(x) = \exp \left\{ ia \int_{x,L}^{\infty} d\xi_\beta \int d^3 z \left[\frac{1}{-\Box} \right] (\xi - z) \partial_\alpha F^{\alpha\beta}(z) \right\}$$

$$\sigma(x) = \exp \left\{ ia \int_{x,L}^{\infty} d\xi^i A_i(\xi) + \left[\frac{\partial_\alpha A^\alpha(x)}{-\Box} \right] \right\} \tag{3.2}$$

We see that σ is gauge invariant. As in the case of μ the renormalized correlation functions of σ are local, in spite of the fact that it explicitly depends on the curve

L [11]. It can be shown [11] that σ creates states bearing a units of the charge corresponding to the conserved current

$$j^\mu = \partial_\nu \left[\frac{F^{\nu\mu}}{(-\Box)^{1/2}} \right] \tag{3.3}$$

The renormalized euclidean correlation functions of σ are given by [11]

$$< \sigma(x)\sigma^\dagger(y) >= Z^{-1} \int DA_\mu \exp \left\{ - \int d^3z \left[\frac{1}{4} F_{\mu\nu} \left[\frac{1}{(-\Box)^{1/2}} \right] F^{\mu\nu} \right.\right.$$

$$\left.\left. +i\ F_{\mu\nu} \left[\frac{1}{(-\Box)^{1/2}} \right] \tilde{C}^{\mu\nu} - \frac{1}{2} \tilde{C}_{\mu\nu} \left[\frac{1}{(-\Box)^{1/2}} \right] \tilde{C}^{\mu\nu} \right] \right\} \tag{3.4}$$

where $\tilde{C}^{\mu\nu} = \tilde{C}^{\mu\nu}(z;x) - \tilde{C}^{\mu\nu}(z;y)$. An arbitrary n-point correlation function would be obtained by the introduction of additional external sources $\tilde{C}^{\mu\nu}(z;x_i)$. The second term in the above expression corresponds to the operator itself and the last one is a renormalization factor which makes the correlation function completely local (independent of the path L). Expression (3.4) was evaluated in [11] giving the result

$$< \sigma(x)\sigma^\dagger(y) >= \lim_{m,\epsilon \to 0} \exp \left\{ -\frac{a^2}{4\pi^2} \left[\ln m|x-y| - \ln m|\epsilon| \right] \right\} \tag{3.5}$$

where m and ϵ are respectively infrared and ultraviolet regulators introduced in order to control the singularities of the euclidean propagator corresponding to (2.6), namely

$$D^{\sigma\lambda} = \left(-\Box \delta^{\sigma\lambda} + \left(1 - \frac{1}{\xi} \right) \partial^\sigma \partial^\lambda \right) \left[\frac{1}{(-\Box)^{3/2}} \right]$$

with

$$\left[\frac{1}{(-\Box)^{3/2}} \right] (x-y) = \lim_{m,\epsilon \to 0} -\frac{1}{8\pi^2} \ln m^2 \left[|x-y|^2 + |\epsilon|^2 \right] \tag{3.6}$$

(ξ is the gauge fixing parameter).

We see that the infrared cutoff m is completely cancelled and we get

$$< \sigma(x)\sigma^\dagger(y) >_R = \frac{1}{|x-y|^{\frac{a^2}{4\pi^2}}} \tag{3.7}$$

where the ultraviolet cutoff was eliminated through the renormalization $\sigma_R = \sigma e^{|\epsilon^2|/2}$. Should we compute a non-neutral correlation function like $< \sigma\sigma >$ we would have

the relative sign in (3.5) reversed, the m factors would no longer be cancelled and the correlation function would be equal to zero.

Another interesting result is the mixed $\sigma - \mu$ correlation function. This was also evaluated in [11], giving the result

$$< \sigma(x_1)\sigma^\dagger(x_2)\mu(y_1)\mu^\dagger(y_2) >_R = \frac{1}{|x_1 - x_2|^{\frac{a^2}{4\pi^2}}} \frac{1}{|y_1 - y_2|^{b^2}} \exp\{A(x_1 - y_1) + A(x_1 - y_2) + A(x_2 - y_1) + A(x_2 - y_2)\} \tag{3.8}$$

where

$$A(x_i - y_j) \equiv n\ iab\ arg(x_i - y_j) \tag{3.9}$$

with $n = 0, \pm 1, \ldots$. The ambiguity of the above correlation function up to $arg(x_i - y_j)$ functions is a reflex of the many possible orderings of operators in the l.h.s. and indicates that σ and μ do satisfy the following dual algebra [11]

$$\sigma(x)\mu(y) = \mu(y)\sigma(x) \exp\{iab\ \arg(\vec{y} - \vec{x})\} \tag{3.10}$$

3.2) The "Sine-Maxwell" Theory

In order to obtain a pure gauge theory with massive vortex excitations in 2+1D, let us proceed as in the case of Sine-Gordon and consider the theory whose vacuum functional is given by

$$Z_R = \sum_{m=0}^{\infty} (-1)^m \frac{(\alpha/2)^m}{m!} \int \prod_{i=1}^{m} d^3 z_i \sum_{\lambda_i} Z_R^{(m)}(z_1, \ldots z_m) \tag{3.11}$$

where

$$Z_R^{(m)}(z_1, \ldots z_m) = < \sigma_{\lambda_1}(z_1) \ldots \sigma_{\lambda_m}(z_m) >_{\alpha=0,R} \tag{3.12}$$

and α is a real parameter which will be later on identified with the fugacity of the gas. In (3.11) and (3.12) $\lambda_i = \pm 1$ and we use the convention that $\sigma_+ = \sigma$ and $\sigma_- = \sigma^\dagger$. The sum $\sum_{\lambda_i}$ runs over all possible configurations of λ's in the set $\{\lambda_i\}$. As we saw in the last subsection, only "neutral" configurations for which $\sum_i^m \lambda_i = 0$ and $m = 2n$, do contribute to (3.11). The $2n$-point correlation function

$$< \sigma_{\lambda_1}(z_1) \ldots \sigma_{\lambda_n}(z_{2n}) >_{\alpha=0,R} = \lim_{m,\epsilon \to 0} \exp\left\{ \frac{a^2}{16\pi^2} \sum_{i,j=1}^{2} n\lambda_i\lambda_j \ln m^2 \left[|z_i - z_j|^2 + |\epsilon|^2 \right] \right\}$$

$$= Z_R^{(2n)}(z_1, \dots z_{2n}) \tag{3.13}$$

is the Boltzmann weight of a three dimensional gas of point particles with a logarithmic interaction. From (3.13) it is clear that the infrared cutoff m is completely canceled because $\sum_i^m \lambda_i = 0$. Since $m = 2n$, the sum $\sum_{\lambda_i}$ in (3.11) just gives an overall factor $\frac{m!}{(n!)^2}$. The self-energies of the particles, which correspond to the $i = j$ terms in (3.13) can be eliminated by renormalizing the fugacity as $\alpha_R = \alpha|\epsilon|^{\frac{a^2}{8\pi^2}}$. The resulting vacuum functional is the grand-partition function of the same gas of point particles with logarithmic interaction, namely,

$$Z_R = \sum_{n=0}^{\infty} \frac{(\alpha_R/2)^{2n}}{(n!)^2} \int \prod_{i=1}^{2n} d^3 z_i \exp\left\{ \frac{a^2}{8\pi^2} \sum_{i\neq j=1}^{2n} \lambda_i \lambda_j \ln[|z_i - z_j| + |\epsilon|] \right\} \tag{3.14}$$

This is almost identical to the two-dimensional Coulomb Gas of point particles [4, 14]. Here, however, the logarithmic interaction is not a Coulomb interaction. There are still ultraviolet divergences in (3.14) caused by the coalescence of p-positive and p-negative charges. The analysis of these divergences can be made exactly by the same method applied to the Coulomb Gas in the 1+1D case [14]. We will have divergences for

$$a^2 = 8\pi^2 \frac{3(2p-1)}{2p} \tag{3.15}$$

corresponding to the coalescence of a neutral p-pole. For $a^2 < 12\pi^2$, the theory is finite because the singularities are integrable. For $12\pi^2 \leq a^2 < 24\pi^2$ we will have the divergences corresponding to neutral p-poles, given by (3.15). These can be eliminated by a multiplicative renormalization of (3.14) [14]. For $a^2 \geq 24\pi^2$, we start to have divergences corresponding to non-neutral configurations in addition to the ones given by (3.15) and we do not know how to eliminate them [14]. We can assert, however, that the theory makes sense for $a^2 < 24\pi^2$. At this point the theory probably undergoes a phase transition analogous to the one of Kosterlitz-Thouless which takes place in the corresponding two-dimensional system [14].

Let us investigate now whether we can associate a lagrangian to the vacuum functional Z_R. For this let us consider the functional integral expressing the m-point σ

correlation function

$$Z_R^{(m)}(z_1,\dots z_m)=Z_0^{-1}\int DA_\mu \exp\left\{-\int d^3z\left[\frac{1}{4}F^{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}+\mathcal{L}_{GF}\right.\right.$$

$$+i\ \sum_{i=1}^{m}\lambda_i\tilde{C}_{\mu\nu}(z;z_i)\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}$$

$$\left.\left.-\ \left[\sum_{i=1}^{m}\lambda_i\tilde{C}_{\mu\nu}(z;z_i)\right]\left[\frac{1}{(-\Box)^{1/2}}\right]\left[\sum_{j=1}^{m}\lambda_j\tilde{C}_{\mu\nu}(z;z_j)\right]\right]\right\} \tag{3.16}$$

where $\tilde{C}_{\mu\nu}$ is given by (3.1). Neglecting the last term which is a renormalization factor, and inserting the unrenormalized Boltzmann weight $Z^{(m)}(z_1,\dots z_m)$ in (3.11) we get

$$Z=\sum_{m=0}^{\infty}(-1)^m\frac{(\alpha/2)^m}{m!}\int\prod_{i=1}^{m}d^3z_i\sum_{\lambda_i}Z_0^{-1}\int DA_\mu \exp\left\{-\int d^3z\left[\frac{1}{4}F^{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}\right.\right.$$

$$\left.\left.+\mathcal{L}_{GF}+i\ \sum_{i=1}^{m}\lambda_i\tilde{C}_{\mu\nu}(z;z_i)\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}\right]\right\}$$

We immediately recognize the sum in (3.11) as an expression of the cossine and therefore

$$Z=Z_0^{-1}\int DA_\mu \exp\left\{-\int d^3z\left[\frac{1}{4}F^{\mu\nu}(z)\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}(z)+\mathcal{L}_{GF}\ +\right.\right.$$

$$\left.\left.\alpha\cos\left[\int d^3z'\tilde{C}_{\mu\nu}(z;z')\left[\frac{1}{(-\Box)^{1/2}}\right]F^{\mu\nu}(z')\right]\right]\right\} \tag{3.17}$$

The theory which corresponds to Z is therefore described by the lagrangian

$$\mathcal{L}_{SM}=\frac{1}{4}F^{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F_{\mu\nu}+\alpha\cos\left[\int d^3z\tilde{C}_{\mu\nu}\left[\frac{1}{(-\Box)^{1/2}}\right]F^{\mu\nu}\right] \tag{3.18}$$

This is the three-dimensional theory which corresponds to the Sine-Gordon. Since it involves the vector gauge field we call it "Sine-Maxwell". In its renormalized form, given by definition by the functional Z_R, Eq. (3.14), it becomes equivalent to a three-dimensional gas of point particles with a logarithmic interaction. As we mentioned before this is perfectly well defined for a certain range of the parameter a, namely $a^2<24\pi^2$.

In the next section, we are going to show that this theory presents some features that are usually associated to the presence of a Higgs field with a symmetry breaking

potential coupled to a gauge field, namely, a non-vanishing vacuum expectation value for the charge bearing operator, in this case σ, and the presence of massive excitations carrying the topological charge, that is, massive vortices.

4) Correlation Functions

4.1) $<\sigma>$

Let us consider firstly the vacuum expectation value of the charge bearing operator σ. As we saw in Section 2, we had $<\sigma>=0$ in the nonlocal Maxwell theory, given by (2.6) because of the infrared divergences appearing in (3.5) and (3.6). In the "Sine-Maxwell theory, we have

$$<\sigma(x)>=\sum_{m=0}^{\infty}(-1)^m\frac{(\alpha/2)^m}{m!}\int\prod_{i=1}^{m}d^3z_i\sum_{\lambda_i}Z_R^{(m)}(x;z_1,\dots z_m) \tag{4.1}$$

where $Z_R^{(m)}(x;z_1,\dots z_m)=<\sigma(x)\sigma_{\lambda_1}(z_1)\dots\sigma_{\lambda_m}(z_m)>_{\alpha=0,R}$ is given by an expression identical to (3.16), except for the exchange

$$\sum_{i=1}^{m}\lambda_i\tilde{C}_{\mu\nu}(z;z_i)\longrightarrow\tilde{C}_{\mu\nu}(z;x)+\sum_{i=1}^{m}\lambda_i\tilde{C}_{\mu\nu}(z;z_i) \tag{4.2}$$

As only neutral configurations can contribute to $<\dots>^{\alpha=0}$ we immediately see that now we must have $m=2n+1$ with n positive and $n+1$ negative λ's. The expression for the appropriate free correlation function appearing in (4.1) is a trivial extension of (3.13) [11]:

$$Z_R^{(2n+1)}(x;z_1,\dots z_n;\overline{z}_1\dots\overline{z}_{n+1})=\lim_{m,\epsilon\to 0}\exp\left\{\frac{a^2}{16\pi^2}\left[2\sum_{i=1}^{n}\ln m^2[|x-z_i|^2+\epsilon^2]\right.\right.$$

$$-2\sum_{i=1}^{n+1}\ln m^2[|x-\overline{z}_i|^2+\epsilon^2]+\ln m^2\epsilon^2+$$

$$\left.\left.+\sum_{i,j=1}^{n}\ln m^2[|z_i-z_j|^2+\epsilon^2]+\sum_{i,j=1}^{n+1}\ln m^2[|\overline{z}_i-\overline{z}_j|^2+\epsilon^2]+-2\sum_{i=1}^{n}\sum_{j=1}^{n+1}\ln m^2[|z_i-\overline{z}_j|^2+\epsilon^2]\right]\right\} \tag{4.3}$$

We immediately see that the infrared cutoff m is proportional to $(2n-2(n+1)+1+n^2+(n+1)^2-2n(n+1))=0$ and therefore completely disappears. Renormalizing α

as before, introducing the renormalized α: $\alpha_R = \alpha\epsilon^{a^2/8\pi^2}$ and inserting the resulting correlation function in (4.1), we get

$$< \sigma(x) >_R = \sum_{n=0}^{\infty} \frac{(\alpha_R/2)^{2n+1}}{(n!)(n+1)!} \int \prod_{i=1}^{2n} d^3 z_i \int \prod_{i=1}^{2n+1} d^3 \overline{z}_i \exp\left\{ \frac{a^2}{8\pi^2} \left[2\sum_{i=1}^{n} \ln|x - z_i| \right.\right.$$

$$\left.\left. -2\sum_{i=1}^{n+1} \ln|x - \overline{z}_i| + \sum_{i\neq j=1}^{n} \ln|z_i - z_j| + \sum_{i\neq j=1}^{n+1} \ln|\overline{z}_i - \overline{z}_j| + -2\sum_{i=1}^{n}\sum_{j=1}^{n+1} \ln|z_i - \overline{z}_j| \right] \right\} \tag{4.4}$$

Since the infrared cutoff m has been completely canceled from the above expression we conclude that $< \sigma(x) >_R \neq 0$. This happens because the extra charge introduced by the operator σ has been neutralized by the "gas charges" in the grand-canonical ensemble. The situation is analogous to the one which occurs with a Higgs field in the presence of a symmetry breaking potential. It would be very interesting to investigate whether this fact would produce a massive state in the gauge field spectrum as in the Higgs mechanism. However, the situation here is not as clear because the mass spectrum of nonlocal theories like (3.18) is quite unusual [13].

4.2) $< \mu(x)\mu^\dagger(y) >$

Let us turn now to the $< \mu\mu^\dagger >$ correlation function. It is clear that

$$< \mu(x)\mu^\dagger(y) >_R = \sum_{m=0}^{\infty} (-1)^m \frac{(\alpha/2)^m}{m!} \int \prod_{i=1}^{m} d^3 z_i \sum_{\lambda_i} < \mu(x)\mu^\dagger(y)\sigma_{\lambda_1}(z_1)\ldots\sigma_{\lambda_m}(z_m) >_{\alpha=0,R} \tag{4.5}$$

The above mixed correlation functions were calculated in [11]. A special case of them is given by (3.8). The neutrality condition is now clearly respected with an equal number of positive and negative gas charges. The fugacity α is renormalized as before. Inserting the general expression for the mixed correlation functions in (4.5) [11] (also according to (3.8)) we obtain

$$< \mu(x)\mu^\dagger(y) >_R = \sum_{n=0}^{\infty} \frac{(\alpha_R/2)^{2n}}{(n!)^2} \int \prod_{i=1}^{2n} d^3 z_i \exp\left\{ -b^2 \ln|x-y| + \frac{a^2}{2} \sum_{i\neq j}^{2n} \overline{\lambda}_i \overline{\lambda}_j \ln|z_i - z_j| \right.$$

$$\left. + \frac{iab}{4} \sum_{i=1}^{2} \sum_{j=1}^{2n} \lambda_i \overline{\lambda}_j A(z_j - x_i) \right\} \tag{4.6}$$

where $A(z_j - x_i)$ was defined in (3.9). From the above expression, we see that

$$\lim_{|\vec{x}-\vec{y}|\to\infty} < \mu(x)\mu^\dagger(y) >= 0 \tag{4.7}$$

indicating that $< \mu >= 0$ which implies by its turn that the μ-operator does create true excitations out of the vacuum. Comparing (4.6) with the corresponding expression in the Sine-Gordon theory [?] we can see that they are identical, except for the dimensionality of the space. On the basis of this observation we should expect that the vortex states created by μ are massive as the corresponding soliton excitations in the Sine-Gordon theory.

5) Conclusion

Starting from a gauge theory in which the vortex operator creates massless states in 2+1D, we made a parallel with the theory of a free massless scalar field in two dimensions, where the quantum solitons have zero mass, and used a gas of point charges construction, in order to obtain a pure abelian gauge theory which in many senses resembles the Sine-Gordon. The quantum vortices become massive in this theory in analogy to the Sine-Gordon solitons. Also there is a symmetry breaking order parameter analog to the vacuum expectation value of a Higgs or Sine-Gordon field. The theory is completely mapped in a three dimensional gas of point charges with logarithmic interaction in the grand-canonical ensemble. The analysis of divergences which was applied to the Coulomb gas in 2D can be carried through straightforwardly in the 3D case and we thereby can conclude that the theory is sensible for a certain range of the coupling constant.

The starting theory which one obtains by turning off the cossine interaction appears in the bosonization of the free massless Dirac fermion field in 2+1D. We therefore should expect that the theory obtained when the cossine interaction is included may be related to the bosonization of the massive fermion field in three dimensional space-time. This point deserves a further investigation. Also an interesting subject wich is presently being investigated is the existence of classical vortex solutions with

a finite energy. These would be the classical counterparts of the massive quantum states created by the vortex operator.

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Figure Captions

Fig.1 - Surface used in the definition of the vortex operator. $L(x)$ is the cut of the function $arg(z-x)$